



**SIGGRAPH2010**  
The People Behind the Pixels

# *Fast Furry Ray Gathering*

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Rhythm + Hues Studios

# Objective

- Render fur convincingly:
  - Lighting interacts with geometry in the scene
  - Objects and environment maps serve as light sources
- Rendering must be fast and art-directable

# Objective

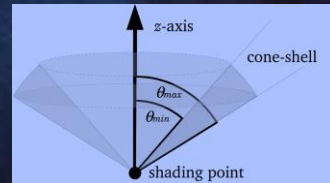
- Our approach: “ray gathering”
  - Sample incident radiance along many directions
  - Weight it by material’s BRDF
  - Determine reflected radiance
- Furry creatures deform → can’t precompute
- How do we do this *efficiently*?



# Overview

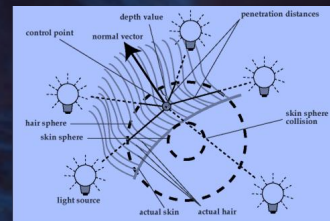
## 1. Hair Scattering Model

- Concise generates gather rays



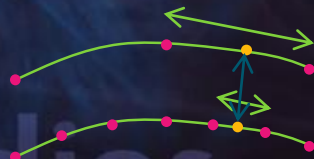
## 2. Volumetric Occlusion Model

- Eliminates ray-hair intersections



## 3. Reflection Cache

- Evaluate fewer shading samples



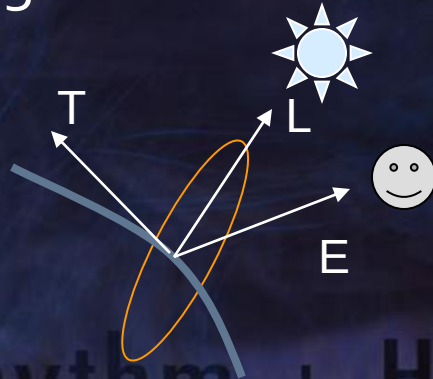
# 1) Hair Scattering Model



- Kajiya-Kay model [SIGGRAPH '89]
  - Treats hair strand as an infinitesimally thin cylinder
  - Consists of separate diffuse, specular submodels
  - Describes reflectance in terms of hair tangent

# 1) Hair Scattering Model: Kajiya-Kay Diffuse Reflectance

$$\begin{aligned}\text{Diffuse} &= \sin(\mathbf{T}, \mathbf{L}) = \text{sqrt}(1 - \mathbf{T} \cdot \mathbf{L}^2) \\ &= \pi \text{ avg}(\max(\mathbf{N} \cdot \mathbf{L}, 0)) \text{ all around } \mathbf{T} \\ &\propto \text{average Lambert reflectance around strand}\end{aligned}$$

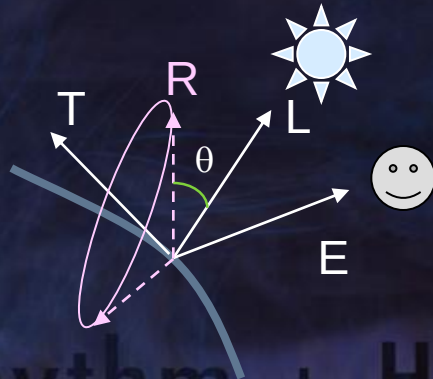




# 1) Hair Scattering Model: Kajiya-Kay Specular Reflectance

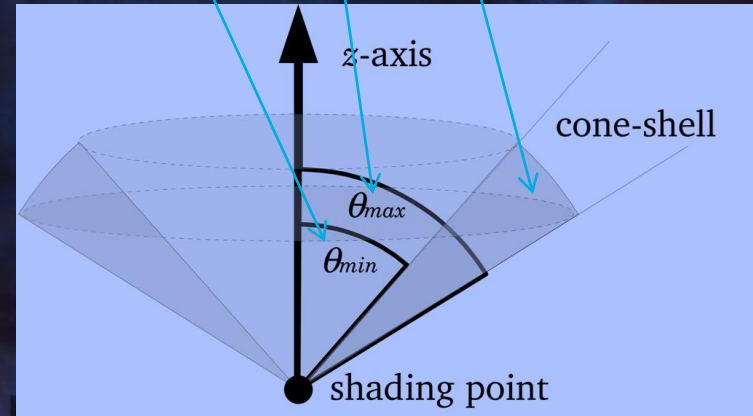
$$\begin{aligned}\text{Specular} &= [\sin(\mathbf{T}, \mathbf{L}) \sin(\mathbf{T}, \mathbf{E}) - (\mathbf{T} \cdot \mathbf{L})(\mathbf{T} \cdot \mathbf{E})]^p \\ &= [\text{sqrt}(1 - \mathbf{T} \cdot \mathbf{L}^2) \text{sqrt}(1 - \mathbf{T} \cdot \mathbf{E}^2) - (\mathbf{T} \cdot \mathbf{L})(\mathbf{T} \cdot \mathbf{E})]^p \\ &= \cos^p \theta\end{aligned}$$

- $p$  is a phong-style exponent



# 1) Hair Scattering Model: Our *Cone-Shell* Model

- Defined by a pair of concentric cones about z-axis, bounded by:  $\theta_{min}$ ,  $\theta_{max}$ 
  - ▣ Solid angle between the cones is the “Cone-Shell”
- Thickness of cone-shell defines blurriness of reflection

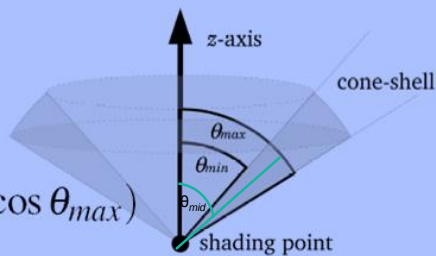




# 1) Hair Scattering Model: Our *Cone-Shell* Model

- U(0,1) pair  $\xi_1, \xi_2$  generates ray in direction  $(x,y,z)$  with BRDF *weight*

$$\begin{aligned}x &= \sqrt{1 - z^2} \cos(2\pi\xi_2) \\y &= \sqrt{1 - z^2} \sin(2\pi\xi_2) \\z &= \cos \theta_{max} + \xi_1 (\cos \theta_{min} - \cos \theta_{max}) \\weight &= z \cos \theta_{mid} + \sqrt{1 - z^2} \sin \theta_{mid}\end{aligned}$$

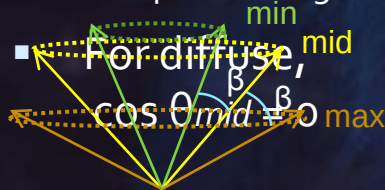


- Intuition for *weight*:
  - cosine of angle ray makes with middle of cone-shell ( $\theta_{mid} = \theta_{(min+max)/2}$ )
  - $= \cos(\theta_{mid} - \theta_{ray})$
  - $= \cos \theta_{mid} \cos \theta_{ray} - \sin \theta_{mid} \sin \theta_{ray}$
  - $= z \cos \theta_{mid} - (1 - z^2)^{1/2} \sin \theta_{mid}$

# 1) Hair Scattering Model: Our *Cone-Shell* Model

- At render time, we express these in terms of

- $E$  = eye vector
- $T$  = tangent vector
- $\beta$  = blur angle radius



- For diffuse,

$\cos \theta_{mid} = 0$

- We taper weight for  $\beta < \pi/2$

$$x = \sqrt{1 - z^2} \cos(2\pi\xi_2)$$

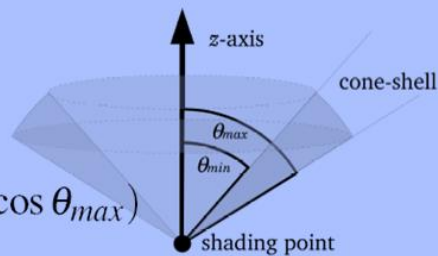
$$y = \sqrt{1 - z^2} \sin(2\pi\xi_2)$$

$$z = \cos \theta_{max} + \xi_1 (\cos \theta_{min} - \cos \theta_{max})$$

$$weight = z \cos \theta_{mid} + \sqrt{1 - z^2} \sin \theta_{mid}$$

$$\cos \theta_{mid} = -\mathbf{E} \cdot \mathbf{T}$$

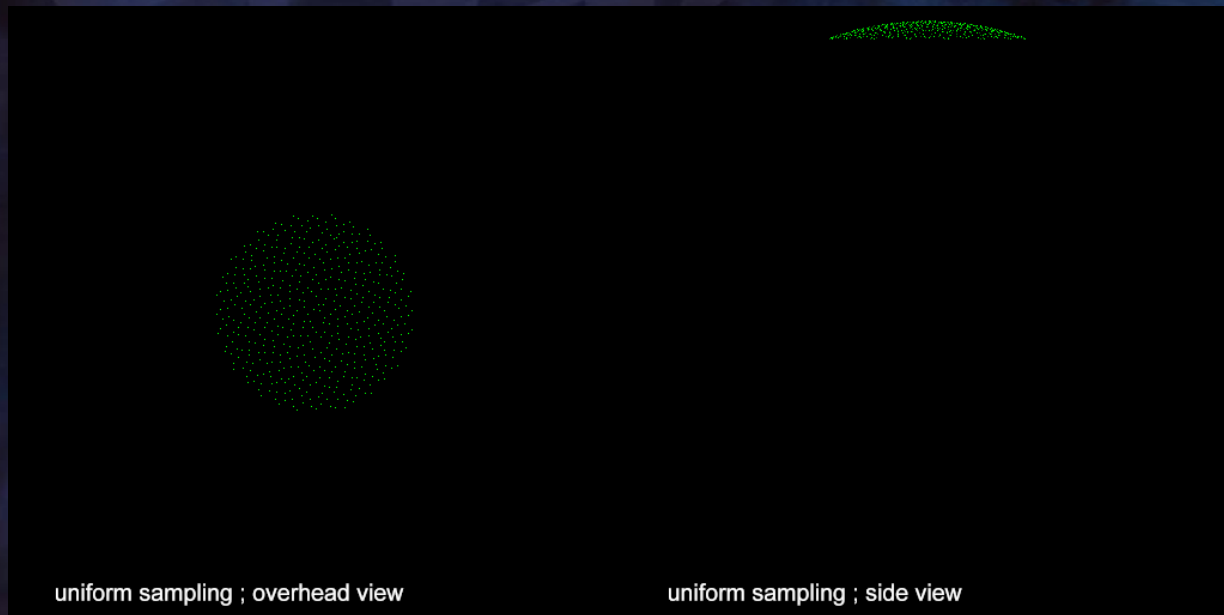
$$\sin \theta_{mid} = \sqrt{1 - \cos^2 \theta_{mid}}$$



$$weight := weight + \frac{\cos \beta}{\cos \beta - 1} (1 - weight)$$

# 1) Hair Scattering Model: Our *Cone-Shell* Model

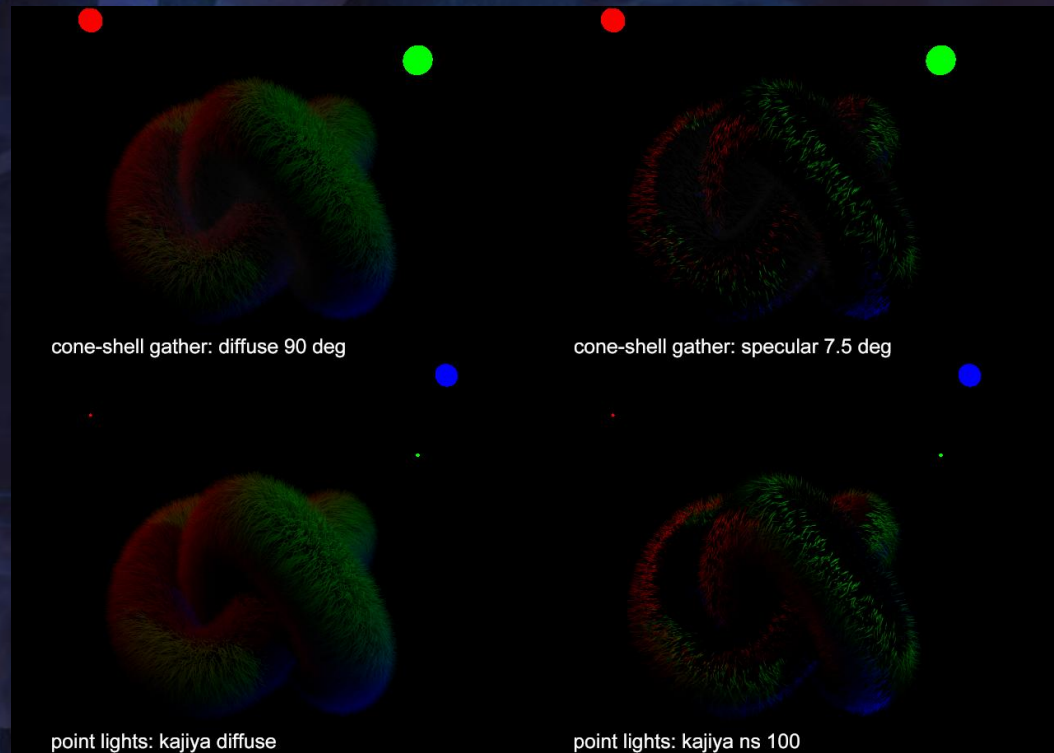
- Specular Cone-Shell (500 rays,  $\beta = 20^\circ$ )





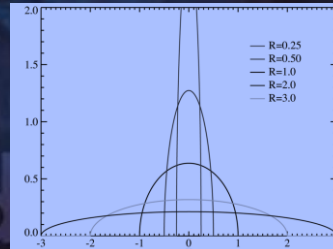
# 1) Hair Scattering Model: Our *Cone-Shell* Model

- Compare:  
Area-lit cone-shell  
VS  
Point-lit Kajiya-Kay
  - Diffuse
  - Specular



# 1) Hair Scattering Model: BRDF Importance Sampling

- IS → Draw samples with probability proportional to BRDF weight
  - Essential for  $\cos^n$  lobe, less for cone-shell
- “ideal” pdf is Wigner Semicircle Distribution
  - Ideal only if incoming radiance is uniform
  - We chose a *raised triangle distribution*



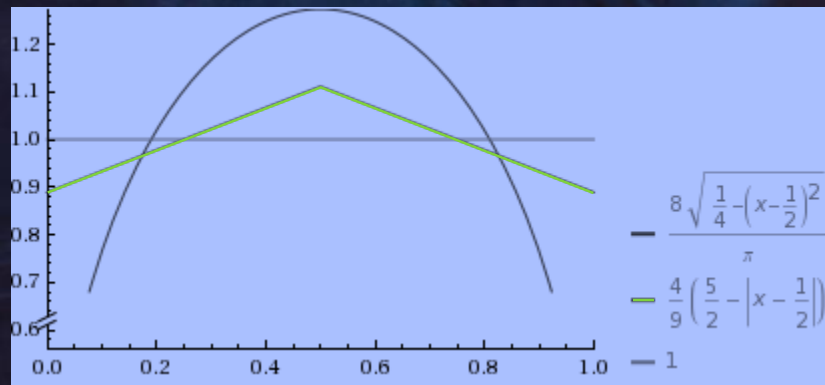
# 1) Hair Scattering Model: BRDF Importance Sampling

- Raised Triangle Distribution:

$$\frac{4}{9} \left( \frac{5}{2} - \left| x - \frac{1}{2} \right| \right) \quad \text{pdf}$$

- How to generate:

$$\xi_1 = \begin{cases} \sqrt{\frac{1}{2}(8+9\xi)} - 2 & : \xi \leq 1/2 \\ 3 - \sqrt{\frac{1}{2}(17-9\xi)} & : \xi > 1/2 \end{cases}$$





# 1) Hair Scattering Model: BRDF Importance Sampling

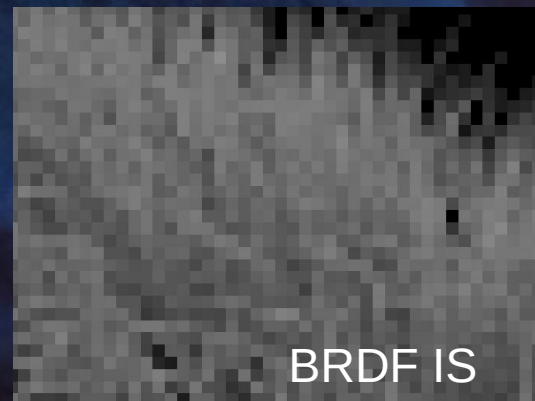
- Diffuse reflection of white sphere (30 rays/pixel)



uniform sampling



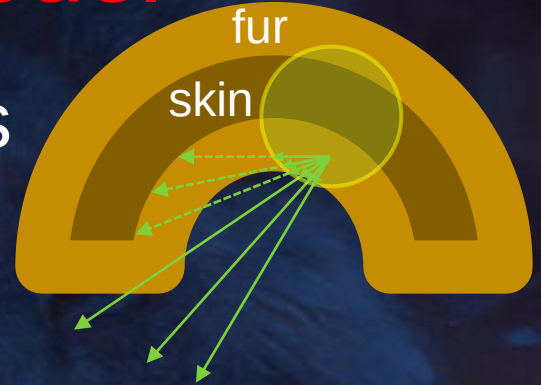
importance sampling



BRDF IS

## 2) Volumetric Occlusion Model

- Goal: omit ray-hair intersections
- Assume hair is short and uniformly covers skin
- Compute each ray's fractional occlusion using volumetric local occlusion model
- Supplement occlusion estimate with accurate ray-traced occlusion to skin geometry

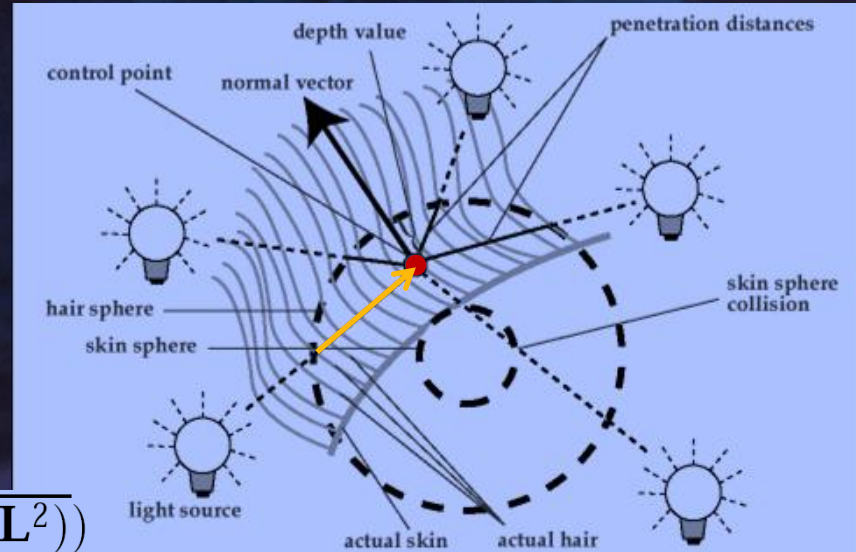


## 2) Volumetric Occlusion Model: Details



- Based on on SIGGRAPH 2004 sketch by Neulander

- Each CP placed in virtual sphere based on  $\mathbf{N}_O$ ,  $h_O$
- Occlusion depends on distance through sphere



$$e^{-\rho(-h_O \mathbf{N}_O \cdot \mathbf{L} + \sqrt{1 - h_O^2(1 - \mathbf{N}_O \cdot \mathbf{L}^2)})}$$



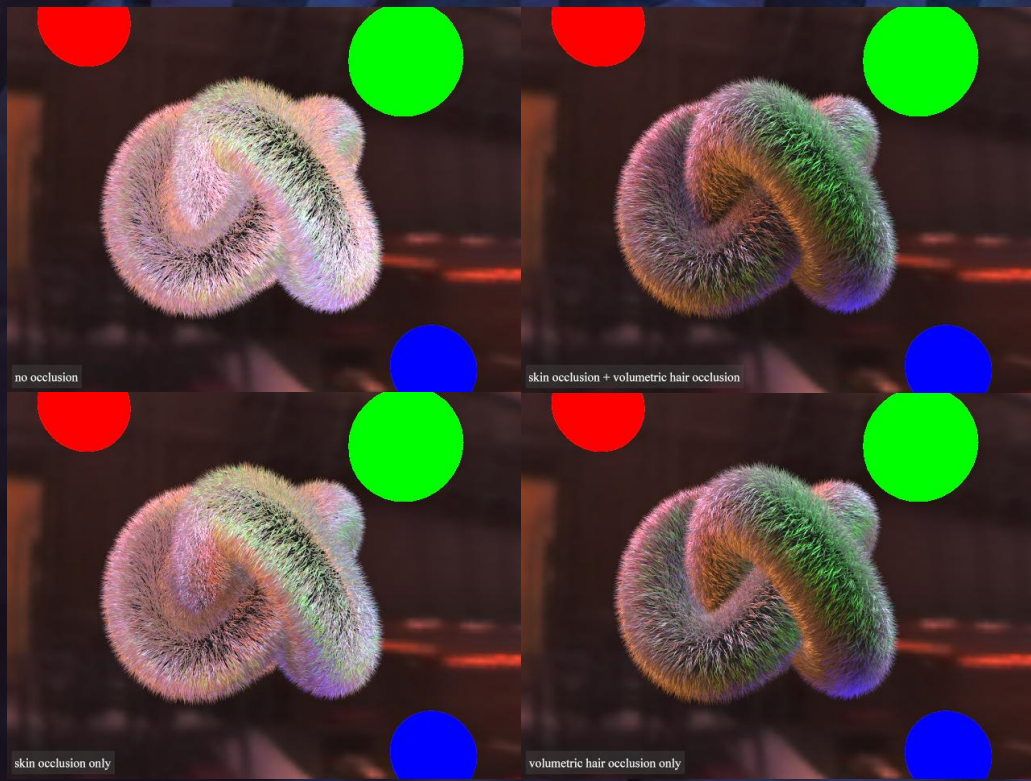
## 2) Volumetric Occlusion Model: Example



skin occlusion + volumetric hair occlusion

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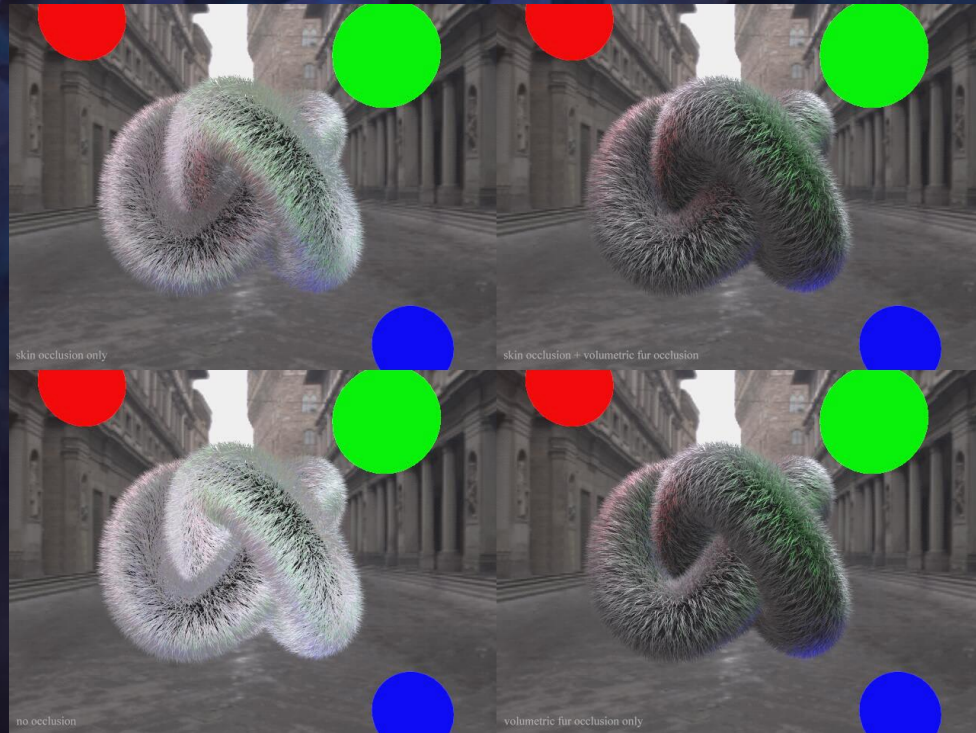
## 2) Volumetric Occlusion Model: Example



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## 2) Volumetric Occlusion Model: Animation



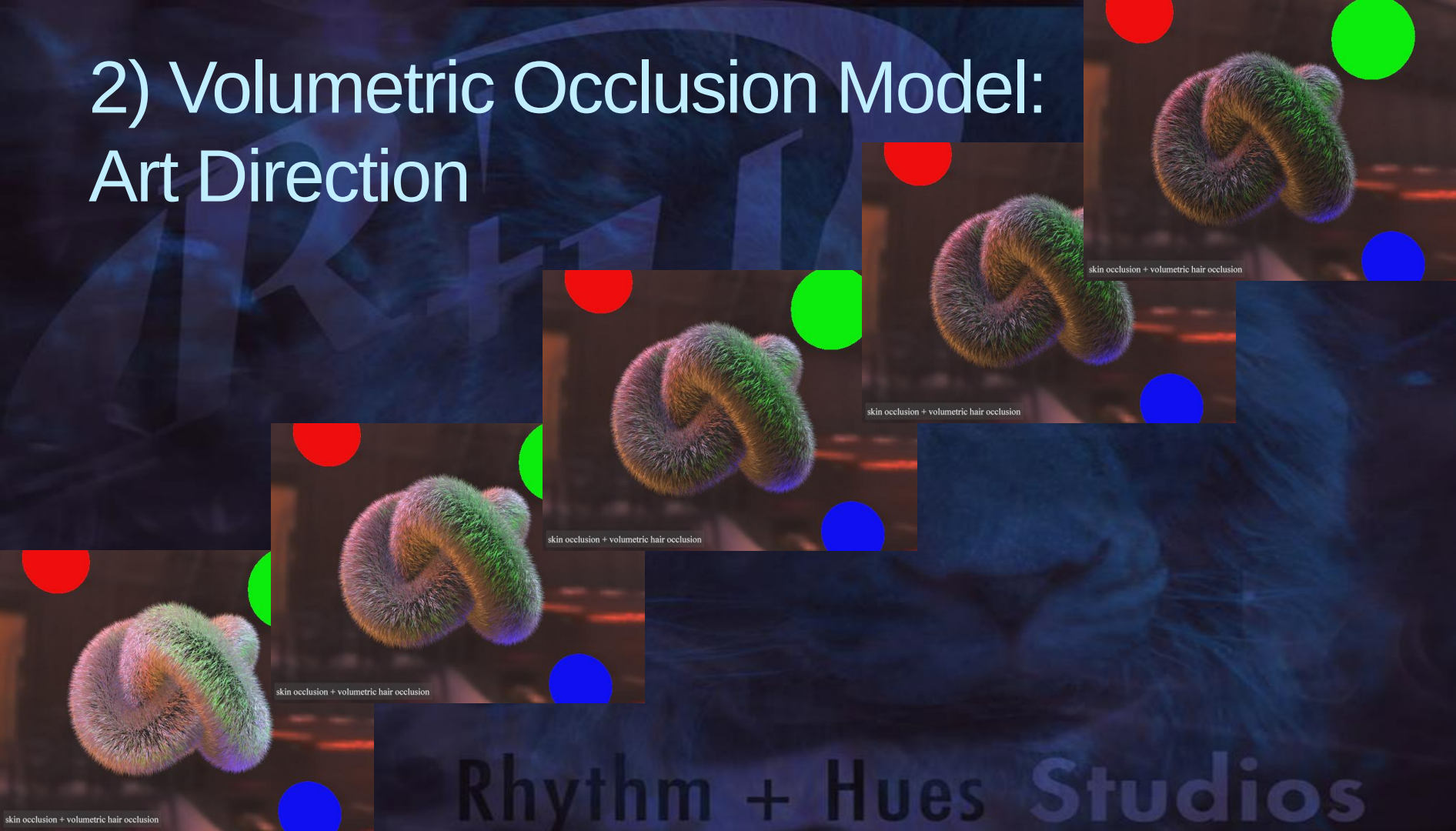


## 2) Volumetric Occlusion Model: Art Direction

- We can alter the apparent fur density:
  - Change exponent  $\rho$
  - Geometry stays the same



## 2) Volumetric Occlusion Model: Art Direction



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### 3) Reflection Cache

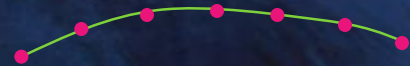
- Evaluate subset of shading requests
  - Cache a sparse set of points, interpolate
- Ward-style irradiance caching doesn't help
  - Irradiance varies abruptly between strands
    - but it varies gradually along any one strand





### 3) Reflection Cache: Our Approach

- Cache reflected radiance at primary rays along each strand
  - Diffuse and specular radiance cached separately
    - at separate user-defined densities
  - Sample count varies with pixel length
  - Caching is suppressed for very short strands



### 3) Reflection Cache: Temporal Coherence

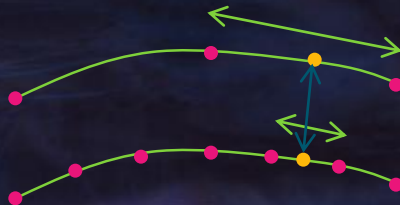


- Problem: Shading may “pop” from frame to frame as sample count changes
- Solution: Compute shading on a fixed  $2^n-1$  lattice for

- $n_1 = \lfloor \log_2 \text{samples} \rfloor$

- $n_2 = \lceil \log_2 \text{samples} \rceil$

- Interpolate between these (similar to mip-map)



### 3) Reflection Cache: Performance Gain



caching off

25 million rays; 260 s



caching on

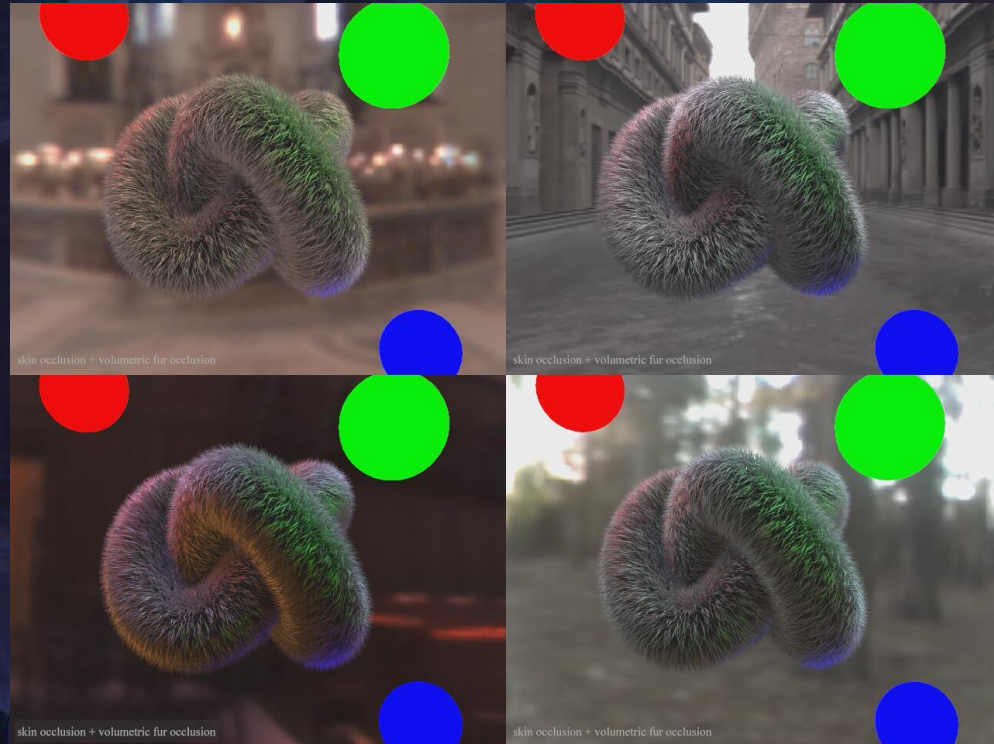
6.2 million rays; 76 s



# 3) Reflection Cache: Results in animation:

## Stats Per Frame

- resolution 640x480
- strands 53k
- 3.8 million rays
- CPU time on Athlon64 5600:  
~ 53s



# Production Results: Knight and Day



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# Future Work

- Approximate multiple scattering using skin-based multi-pole model
- Adaptively modulate cache density based on
  - Hair curvature
  - Perceptual metrics of color variation



# Thank You

- Thanks to Rhythm & Hues
  - Josh Bryer, Ray Chih, Ryan Gillis, Toshi Kato, Dan Lazarow, Mike Sandrik
- Slides and abstract available on:  
[www.rhythm.com/~ivan/pubs.html](http://www.rhythm.com/~ivan/pubs.html)

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